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THE LUMINOSITY SCALE OF CEPHEID VARIABLE STARS: A REVISION

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THE LUMINOSITY SCALE OF CEPHEID VARIABLE STARS: A REVISION

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ABSTRACT

The period-luminosity-color relation for classical Cepheids is discussed in terms of a recent redeterminations of the distance moduli of open clusters with Cepheid members. The distance moduli of eight clusters with well established Cepheid members were determined from Stromgren four-color and Hβ photometry of the B stars. Possible sources of systematic errors in these distance moduli are discussed. Zero points are derived from these data for the period-luminosity-color (PLC) relation on the *UBV* system and the period-luminosity relations for near-infrared magnitudes. The present relations yield absolute magnitudes which are fainter by 0.4–0.6 mag than those now commonly used. The source of this discrepancy is discussed. These lower luminosities cause the pulsational masses of the Cepheids to be less than the evolutionary masses by up to a factor of 2.5 but bring them into good agreement with the masses of beat and bump Cepheids. Thus, the predictions of the pulsation theory are made more consistent but the Cepheid mass discrepancy is reintroduced. This revision of the Cepheid luminosity scale will directly affect distances of galaxies based on the Cepheids, but its effect on the Hubble constant depends on how much weight was placed on the Cepheids in the establishment of the distance scale in the Local Group. A comparison of distance moduli for the Large Magellanic Cloud based on various indicators with those for the Cepheids from the present calibration shows no serious discrepancy.

Subject headings: cosmology — galaxies: Magellanic Clouds — stars: Cepheids — stars: pulsation

I. INTRODUCTION

The Cepheid variable stars are basic to two important areas of modern astronomy, the establishment of the cosmic distance scale and the study of stellar interiors through pulsation theory. In the calibration of the distance scale their usefulness arises both from the fact that they are sufficiently luminous to be observed in other galaxies and from the fact that the luminosities can be accurately inferred from the periods. With the advent of the Space Telescope it will be possible to observe Cepheids in galaxies as distant as the Virgo Cluster, and their usefulness will be enhanced accordingly. However, their application will rely, as it does now, on the calibration of the luminosities using local Cepheids.

The role of Cepheids in the study of stellar interiors arises from pulsation theory which has been developed highly in recent years. The models produced by this theory have shown that the period of a Cepheid is determined largely by its mass and radius. Thus, if the radius can be determined observationally, it is possible to infer the mass. When the radius is calculated from the luminosity and effective temperature, the resulting mass is referred to as the pulsational mass or M_Q. Originally pulsational masses were found to be smaller than masses predicted by stellar evolution theory, M_{ev}, but this discrepancy has been largely removed by adjustments in the luminosities and temperatures (see A. N. Cox 1979, 1980 for a recent discussion and references to previous work). Other mass estimates can be obtained from period ratios of multimode Cepheids (referred to as beat masses, $M_{\rm beat}$) and the presence of bumps in the light curves (referred to as bump masses, $M_{\rm bump}$). These are generally smaller than the evolutionary masses if standard models are used. Many investigators have felt that this discrepancy resulted from problems with the beat and bump masses, and a variety of solutions have been suggested involving such things as helium-enriched envelopes (Cox *et al.* 1977; Cox, Michaud, and Hodson 1978; Cox and Hodson 1978; Cox, King, and Hodson 1978; Cox, Hodson, and King 1979), tangled magnetic fields (Stothers 1979), contamination of radial pulsation by nonradial modes (J. P. Cox 1980), rotation (Deupree 1978), convection (Cogan 1977), and nonlinear effects (Cox, Hodson, and King 1978). At present, it seems fair to say that the resolution of this problem is not known for certain. Clearly, since part of the discrepancy involves the pulsation mass which depends on the luminosities and temperatures of the Cepheids, it is important to make every effort to ensure that the basic observational parameters of the Cepheids are well established.

Although considerable effort has been expended to obtain a reliable luminosity scale for the Cepheids, there is reason to believe that the commonly used luminosities may be in error by significant amounts (see Schmidt 1981*b* for a preliminary discussion). During recent years the present author has been engaged in observational studies of open clusters with Cepheid members in order to clarify this situation. That study consisted of observations in the *uvby*-Hβ system of the B stars in the clusters. This type of photometry is capable of yielding the luminosities and intrinsic colors of such objects. These are then used to obtain the cluster reddenings and distances. The data on the individual clusters are now published, and the present paper is intended to discuss the question of the Cepheid luminosities based on these data and attempt to reconcile the various calibrations which have been carried out previously. The implications of the new luminosity scale will also be discussed briefly.

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II. THE CALIBRATING CLUSTERS AND THEIR DISTANCE MODULI

There has been considerable interest during the past few years in increasing the number of Cepheids available to calibrate the period-luminosity relation. Many clusters and associations with possible Cepheid members have been considered; there are numerous cases where the membership is possible but not fully established. Some investigators have felt it best to utilize these dubious cases so as to increase the number of calibrators. In this study we will take the opposite attitude: we will use only a limited number of calibrating Cepheids for which both the color excesses and luminosities are reasonably well established. Although this limits the number available, it is safer in that spurious values are not introduced into the fitting procedure.

We have not observed any associations with Cepheid members. Since associations are large, they contain many nonmembers within their boundaries, and the membership of any individual star in a given association is difficult to establish. Often membership is assumed because the luminosity of the Cepheid agrees reasonably with expectation. Even if the Cepheid is a bona fide member, the size of the associations is such that the reddening varies considerably over the region and it is not possible to accurately infer it at the location of the Cepheid.

We have also omitted V367 Sct, V810 Cen (HR 4511), and CPD $-57°7400$A from our calibration. V367 Sct is a likely member of NGC 6649 and is of special interest because it is the longest period beat Cepheid and the only such star known in a cluster. Unfortunately, this cluster is very distant ($V-M_V = 15.5$) and heavily reddened [$E(B-V) = 1.5$] (van den Bergh and Madore 1975). As a result the B stars are too faint to observe in the four-color system, particularly in the u filter, without an inordinate amount of large telescope time. V810 Cen is a member of the cluster Stock 14. It was not included in this program because the nature of this star is not certain. It may be more appropriately regarded as a pseudo-Cepheid than a true Cepheid (Eggen 1983*b*). Eggen (1983*a*) showed that the star CPD $-57°7400$A in NGC 6067 is an 11 day Cepheid. However, its absolute magnitude is much too faint for its period if it is a cluster member, and Eggen concluded that it was a member of a background association.

The clusters containing Cepheids which we will use for calibration are listed in Table 1. Each of these clusters has been observed photometrically on the $uvby$-Hβ system for the purpose of determining the distance and reddening. The source of the photometry is in the last column of the table. The distance moduli in column (2) and the color excesses in column (4) were obtained from those data using the calibration of Crawford (1978) for B stars. Some of the distance moduli differ slightly from those given in the original papers. This is due to the use of $A/E(b-y) = 4.28$ (Crawford and Mandwewala 1976) in the present calculations. The distance moduli given in column (7) were obtained from the calibration of Eggen (1980). The standard deviations in columns (3) and (9) were calculated from the scatter among the various stars used in the determination and should give a good estimate of the internal accuracy. In many of these clusters there is significant variation of the extinction across the cluster area and it was necessary to interpolate the reddening to the location of the Cepheid. When this was the case, an estimate of the uncertainty was made, and this is indicated; when no error is cited, it indicates that the extinction across the cluster was constant. The clusters included in this program are often in crowded fields, and it was necessary to eliminate nonmembers from the sample. In column (5) we list the number of stars used in calculating the distance moduli and color excesses, while column (6) gives the number which were observed but rejected from membership. This point will be discussed in detail below.

For comparison we also give distance moduli for these clusters based on spectral types and main-sequence fitting. The spectral types are listed in the photometric papers, and they can be consulted for the original references. The calibration of Turner (1980) was used to derive the distances. Crawford's (1978) mean colors for various spectral types were used to obtain the color excesses. Only stars which were taken to be cluster members from the four-color studies were included. The two values given for M25 were obtained with two different sets of spectral types. Many distance moduli have been published for these clusters based on main-sequence fitting. In the table we have listed those from the compilation of Fernie and McGonegal (1983). For stars in common with Sandage and Tammann (1969) the Fernie and McGonegal distance moduli average 0.19 mag larger, while they are 0.29 mag larger on the average than those of de Vaucouleurs (1978*a*). The former distance moduli are based on the old Hyades distance, and increasing it to presently favored values will bring the Sandage and Tammann distance scale into closer agreement with that of Fernie and McGonegal. Thus, the values used in Table 1 are representative of the distance moduli used commonly for these clusters in recent years.

The properties of the calibrating Cepheids are listed in Table 2. The errors listed in column (7) include both the uncertainty of the distance modulus and that of the absorption derived from $E(b-y)$. It should be noted that, except for second order effects due to variable extinction, the absorption cancels out since the distance moduli were based on the absolute magnitudes of B stars in the clusters. In correcting the colors for reddening we encounter difficulties with bandwidth effects. We have dealt with this by first converting the color excesses from Table 1 to the UBV system using $E(b-y) = 0.777\ E(B-V)$ (Crawford and Mandwewala 1976) which is appropriate to B stars. We have used the factors given by Fernie (1963) to obtain the reddenings appropriate to the Cepheids. This introduces some uncertainty for stars as reddened as some of the Cepheids. Ideally we should use $(b-y)$ colors in the calibration and avoid bandwidth effects. However, there is no photometry on that system for most of the stars in Table 2, so for the present we will have to do the best we can with available data.

III. SYSTEMATIC ERRORS IN THE DISTANCE MODULI

Since we seek to establish a zero point for the period-luminosity-color (PLC) relation, it is important to consider any systematic errors which might be present in the determination of the cluster distance moduli. In this section we will discuss a number of possible sources of error and attempt to estimate how serious they might prove to be.

a) The Number of Stars Observed

The number of stars observed in each cluster ranges from 5 to 39. However, when stars with photometric peculiarities and apparent nonmembers are removed, the number which contribute to the mean distance modulus ranges from 4 to 27. This is fewer than are generally used in main-sequence fitting studies. The number is mainly limited by the B star population in the clusters and in a few cases by the faintness of the stars.

TABLE 1

CALIBRATING CLUSTERS

	Crawford Calibration					Eggen		MK Spectral Types				Main Sequence Fitting		
Cluster (1)	$V_0 - M_v$ (2)	σ (3)	$E(b-y)$ (4)	Stars Used (5)	Stars Rejected (6)	$V_0 - M_v$ (7)	σ (8)	$V_0 - M_v$ (9)	σ (10)	$E(b-y)$ (11)	No. of Stars	$V_0 - M_v$ (12)	$E(b-y)$ (13)	Source of Four-Color Photometry
NGC 129	11.01	0.19	0.44 ± 0.04	11	11	10.95	0.19	9.7	0.6	0.45	5	11.35	0.37	Schmidt 1980*b*
Anon (CV Mon)	10.9	0.3	0.57 ± 0.04	5	5	11.1	0.3	...	...	...	...	11.50	0.57	Schmidt 1983
Ru 79	11.4	0.3	0.53 ± 0.05	6	3	11.2	0.3	...	...	...	...	11.95	0.56	Schmidt 1983
Ly 6	10.6	0.3	0.98	4	1	10.4	0.3	...	...	...	...	11.77	0.94	Schmidt 1983
NGC 6087	9.59	0.09	0.13	11	5	9.90	0.08	8.8	0.3	0.12	7	9.95	0.16	Schmidt 1980*a*; Eggen 1980
M25	8.76	0.10	0.40 ± 0.02	27	12	9.00	0.11	9.7, 7.9	0.2	0.39	9, 15	9.21	0.36	Schmidt 1982*a*
NGC 6664	10.69	0.11	0.55 ± 0.02	11	4	10.75	0.14	9.7	0.6	0.53	3	11.15	0.42	Schmidt 1982*b*
NGC 7790	12.05	0.13	0.40	13	3	12.18	0.14	12.4	0.3	0.40	6	12.85	0.43	Schmidt 1981*a*

TABLE 2

CLUSTER CEPHEIDS

Cepheid (1)	Cluster (2)	Period (days) (3)	$\langle V \rangle$ (4)	$\langle B \rangle - \langle V \rangle$ (5)	$\langle M_v \rangle$ (6)	σ (7)	$\langle B \rangle_0 - \langle V \rangle_0$ (8)
EV Sct	NGC 6664	3.09	10.13	1.15	−2.92	0.14	0.47
CE Cas b	NGC 7790	4.48	10.99	1.12	−2.77	0.13	0.66
CF Cas	NGC 7790	4.88	11.11	1.21	−2.65	0.13	0.75
CE Cas a	NGC 7790	5.14	10.92	1.20	−2.84	0.13	0.74
CV Mon	Anon	5.38	10.30	1.35	−3.0	0.35	0.69
CS Vel	Ru 79	5.90	11.70	1.34	−2.0	0.37	0.73
U Sgr	M25	6.75	6.71	1.11	−3.76	0.13	0.64
DL Cas	NGC 129	8.00	8.94	1.20	−3.95	0.26	0.68
S Nor	NGC 6087	9.75	6.41	0.95	−3.75	0.09	0.80
TW Nor.........	Ly 6	10.79	11.67	2.00	−3.1	0.30	0.88

No attempt has been made in the photometric studies to extend the observations to fainter magnitudes because this would require the inclusion of A stars. It appears that there are difficulties with the calibration for those stars which are possibly related to rotation (Crawford 1979; Schmidt and Forbes 1984). Their inclusion would add little to our confidence in the resulting distances.

In spite of the relatively small number of stars in each cluster, the internal errors in the distance moduli listed in Table 1 are mostly smaller than those generally achieved in main-sequence fitting. Therefore, because the precision of the distance modulus for an individual star is quite good, the values given for the clusters are of an accuracy which is adequate for the purposes of calibrating the luminosity scale.

b) Reduction of the Photometric Indices to the Standard System

It is important that the photometric indices be accurately on the standard system in order to make the application of the calibrations valid. One check which can be performed is to compare the present photometry with the results of another investigator. The only such data available are those of Eggen (1977, 1980) for NGC 6087 and for stars near l Car which were observed for possible inclusion in this program (Schmidt 1980*a*). The mean difference between the two data sets in the sense (present − Eggen) is 0.004 ± 0.004 mag (standard deviation for one star) for $(b - y)$, 0.001 ± 0.008 mag for m_1, and 0.000 ± 0.009 mag for c_1. Only m_1 shows any evidence for a very small color dependence. The β index is more critical, and we find a systematic difference of 0.004 ± 0.007 mag for it. There is a color dependence such that the difference is close to 0.01 mag for stars with $\beta = 2.6$ but averages 0.002 for stars with indices greater than 2.68. Most of the stars observed in the clusters have indices greater than that amount. From these comparisons we conclude that since the Eggen photometry and that of the present program are completely independent, the photometry is accurately on the four-color and Hβ system.

A further concern arises from the fact that some of the clusters observed here are heavily reddened and some of the stars are redder than the usual standards. To circumvent this problem we have used a set of reddened B stars as secondary standards. These will be described in a separate paper. For the present purposes we should consider how well the reddened B stars have been placed on the standard system. This point was discussed in the paper on NGC 6664 (Schmidt 1982*b*), and only the main points will be repeated here. To test whether the color scale had been reproduced accurately, the color excesses of 11 of the standards were obtained both from the four-color photometry and from the spectral types and *UBV* photometry. A comparison showed that the four-color photometry produced reddenings in agreement with those from the spectral types for values up to $E(b - y) = 1$. This indicates that the color scale is on the standard system for reddenings as large as any in the present program.

In the case of the β filter, Schmidt and Taylor (1979) showed that a color term is sometimes present. We have measured the response functions of the filters used after each observing run. Following the methods of Schmidt and Taylor, the color term was then included in the reductions. A comparison of the β indices obtained during different runs with different filters failed to show any evidence of systematic differences. We therefore conclude that the color term has been correctly taken into account and no significant error should arise from this source.

c) Nonlinearities in the Photometry

Because of such effects as dead time in the electronics and photomultiplier nonlinearity, it is possible for magnitude-dependent errors to occur in photoelectric photometry. In the observations care was taken to avoid these effects, but we can also make a few *a posteriori* checks.

The β index is based on the ratio of fluxes through a broad-band and a narrow-band filter. The count rates through the two filters differ by factors of about 6 to 10. Thus, the ratio is sensitive to nonlinearities, and the distance moduli derived from Hβ photometry can be affected. Since the clusters in this program are at very different distances, the stars in them are of quite different magnitudes. Therefore, magnitude-dependent effects are possible between various clusters. In comparing the present distance moduli with those from main-sequence fitting, we find that for all except two of the clusters, Ly 6 and NGC 7790, the two moduli differ by almost the same amount. Of this group, M25 is the nearest and the stars used to determine its distance are mostly between $V = 8$ and $V = 11$. The most distant of that group is Ru 79, which has stars between magnitudes of 13 and 14. Since the comparison of the distances of these clusters to those obtained from *UBV* photometry shows good consistency, we conclude that there is no serious magnitude effect.

The data for M25 are the most extensive of any cluster on our program and span a range of 4 magnitudes. While the stars in M25 are not nearly as faint as some in other clusters, a smaller telescope was used to observe them and the count rates ranged from near the maximum allowable for the photocell to very low rates for the faintest stars. The distance modulus for the stars brighter than $V = 9.5$ in M25 has an average value of 8.73 while the value obtained from stars fainter than $V = 10$ is 8.74. Both of these agree well with the mean for all the stars, 8.76, and this agreement supports both the calibration and the linearity of the photometry.

d) Rejection of Stars

In determining the mean distance moduli, some stars have been omitted. This is justified on the grounds that we must expect some field stars to appear in our sample even though obvious nonmembers were not observed. Additionally, some stars show peculiarities which affect their four-color or Hβ indices in a way that makes the application of the calibrations inappropriate. The rejection of stars was based largely on whether the distances, color excesses, and apparent ages of individual stars were near the mean of the majority of stars. The number of stars rejected in each cluster are listed in Table 1, and it can be seen that it ranges up to 50% of the observed stars.

To test whether the rejection introduces any bias, the distance moduli of the clusters were recalculated with a much less strict criterion for rejecting stars: any star with a modulus within 2 magnitudes of the mean was accepted as a member. This resulted in between one and five additional stars being included, and any which are rejected on this basis are clearly so deviant that there is no justification for their inclusion. Of the eight clusters, there were only two for which the mean distance modulus was changed by more than the errors listed in Table 1. The modulus for NGC 129 decreased by 0.28 mag while that of M25 increased by 0.13 mag. Since the original reasons for rejecting stars were justifiable and since the much more liberal criterion produced little change, we conclude that the rejection of stars should have no important effect on the moduli.

e) The Absolute-Magnitude Calibration

A discussion of the absolute-magnitude calibration of the intermediate-band photometry is beyond the scope of this paper. However, since we are interested in using it to establish the zero point of the Cepheid PLC relation, it is of direct importance. A few brief comments are in order.

Eggen (1974) has constructed a calibration which is similar in form to that of Crawford but was based on somewhat different data and has different evolutionary correction factors. The mean distance moduli of the clusters using the Eggen calibration are listed in column (7) of Table 1. It can be seen that the Eggen calibration produces distances which are mostly larger than those from the Crawford calibration. The average difference is 0.06 mag for all the clusters or 0.14 mag if the three with the largest standard errors are omitted. Thus, the two calibrations are consistent at the level of the internal errors. While not a strong test, this comparison at least reveals no serious problems.

Balona and Shobbrok (1984) have recently derived a new calibration for the intermediate-band photometry. This calibration is based on more data than were available to Crawford and should thus be an improvement. In particular, they were able to carry out the entire calibration process without using any association stars. An application of this calibration to the data for the clusters of Table 1 results in absolute magnitudes which average 0.05 mag brighter than those given in Table 2. If the two most discrepant clusters are omitted, the difference is 0.07 in the opposite sense. Thus, this new calibration gives distance moduli for the clusters and luminosities for the Cepheids which are in good agreement with those obtained from the Crawford calibration.

A concern in using the absolute magnitude calibration is the evolution correction. The data for M25 are the most extensive of the clusters in Table 1 and can be used to test this point. All the stars brighter than $V = 9.5$ are evolved and have a mean distance modulus of 8.73 ± 0.3 (four stars). On the other hand, those with c_1 greater than 0.62 are apparently near the zero-age main sequence (see Fig. 1 of Schmidt 1982*a*) and have a mean modulus of 8.78 ± 0.12 (16 stars). These values both agree well with the mean for the cluster, 8.76. Thus, it appears that the evolution correction in the Crawford calibration has worked well in M25.

f) Emission-Line Stars

Although known Be stars were generally omitted from the observing list or at least from the calculation of the distance moduli, there are no doubt cases of undiscovered emission-line stars in our sample. The presence of emission in the Hβ line will reduce the index and give rise to absolute magnitudes which are too bright. While severe cases result in distance moduli so large that they obviously should be omitted, mild cases will escape detection and will bias the average distances to larger values. To remove such stars, Hα photometry should be undertaken following the methods of Feinstein (1974). However, for the present, we can comment that the removal of any Be stars from our list will further lower our moduli and thus aggravate the disagreement with the previous distance scale.

g) The Presence of Binaries

If a B star has a binary companion, it will affect the inferred distance modulus by contributing light to the V magnitude and by contaminating the β index with the value for a fainter star. The latter will either increase or decrease the index, depending on the spectral types of the two stars. Most of the stars in our clusters are middle to late B stars, and only a companion which is either a later B star or an A star will be bright enough to have an appreciable effect. Such stars have stronger hydrogen lines than the B stars. The tendency will then be for a companion to increase the β index and make the inferred absolute magnitude too faint. Thus, both the contamination of the V magnitude and the contamination of the β index will tend to cause the inferred distance to be too small. Calculations were carried out for B5 V and B7 V stars with various types of main-sequence secondaries. It was found that the largest change in the inferred distance would occur for a secondary only slightly fainter and later than the primary. The largest effect is about 0.8 mag. As the companion becomes later in spectral type and fainter, the effect becomes smaller.

Although there is too little information on the presence of binaries in these clusters to properly account for their effects, a couple of points can be made. First, in the calibration of the absolute magnitudes no binaries were excluded. Thus, insofar as the stars in the present sample contain about the same proportion of binaries as those in the calibrating clusters, their effect should cancel out. Second, Anthony-Twarog (1982) examined the four-color and Hβ data for the Orion association. In that case there are studies which enabled her to eliminate suspected binaries from the sample. She found that for the various subgroups in the association the inferred distances with and without the suspected binaries differed by between

0.01 and 0.17 mag. Thus, we should probably not anticipate any effects from binaries which are larger than the internal errors of our moduli.

h) Rotation

Because stellar rotation changes the effective gravity of a star, it is a potential source of errors in absolute magnitudes derived from photometry. Since we have little rotational velocity data for the stars in this program, we cannot test for any such effects directly. However, Crawford (1978) and Anthony-Twarog (1982) found no systematic effect in the Pleiades or in the Orion association. They concluded that effects of rotation cancel out in the presence of random orientations.

i) Atmospheric Composition

Since it is known that there is a range of atmospheric composition among various clusters (see Nissen 1979 for a summary) we should inquire whether this has an appreciable effect on the distance moduli inferred from Hβ indices. Since we are considering B stars, we should expect little effect from the metal abundances. This is borne out by the calculated Hβ indices (Schmidt 1979) for models with various metallicities: lowering the metal content of the atmosphere by a factor of 100 changes the index by only 0.003 mag.

Stars of the type used in this study are in a region of the color-magnitude diagram in which helium weak stars are common. In fact, a study of M25 and NGC 129 by Schmidt (1978) showed the presence of such stars. Thus, we must question whether this is a possible source of uncertainty in our distance scale. Unfortunately, a thorough study of the effects of helium abundance on the four-color and Hβ photometry is not possible at present. Synthetic Hβ indices (Schmidt 1979) were found to be relatively insensitive to atmospheric helium; a reduction of the number fraction of helium from 0.1 to 0.05 changes the β index by less than 0.007 mag. This corresponds to a change in the inferred absolute magnitude of less than 0.1 mag. Thus, we should expect no significant error if the helium-weak phenomenon is an atmospheric effect as is generally thought.

An empirical test of the effects of helium can be made using the Sco-Cen association in which Nissen (1974) studied the helium abundance. There are six helium-weak stars for which Hardie and Crawford (1961) or Glaspey (1971) obtained Hβ photometry. A plot of the β indices against the V magnitudes for these stars and the apparently normal stars shows no separation. Additionally, for the four helium-weak stars with both four-color and Hβ indices, we obtain a distance modulus of 5.63 ± 0.21 while the apparently normal stars yield a value of 5.85 ± 0.14. Thus, within the errors there is no difference and the He-weak phenomenon should have no significant effect on the distances. Unfortunately, more stars are needed to make this empirical test stronger. The question should be studied further, both for the present purpose and for other applications of the intermediate-band photometry.

j) Primordial Composition

The primordial interior composition affects both the gravities and the luminosities of main-sequence models. A range in composition occurs among Population I stars, so some error might be introduced in the derived distances from this source. Since the clusters with Cepheids are nearly the same age as the α Per and Pleiades clusters which were used for the four-color and Hβ absolute magnitude calibration, we might expect the effect to be minimal and random. However, we will attempt to estimate the possible errors from this source. To do this, we will use the zero-age models of Mengel *et al.* (1979) for various compositions.

The effect of helium abundance can be assessed by referring to the models with $Y = 0.3$ and $Y = 0.4$. This change makes no appreciable change in the gravity, but the more helium-rich models are 0.15 mag brighter for temperatures appropriate to late B stars. Since most clusters seem to have nearly the same helium abundance with only a few differing by as much as 0.1 in Y, we take this to be an estimate of the maximum uncertainty introduced by helium abundance into our distance moduli.

The metallicity affects both the gravity (and thus the Hβ index) and the luminosity at a given temperature. Increasing the metals from $Z = 0.01$ to $Z = 0.04$ (a change of 0.6 dex) causes the gravity to drop by 0.17 in the logarithm, and this will cause a decrease in the β index of 0.014 and the inferred absolute magnitude will be 0.2 mag brighter. On the other hand, the increased metal abundance will increase the luminosity by 0.5 mag, and the distance modulus will be overestimated by 0.3 mag when both effects are combined. Since the range in metals is perhaps ± 0.1 dex (Nissen 1979), we conclude that errors from this source are likely to be less than 0.05 mag.

IV. THE PERIOD-LUMINOSITY-COLOR RELATION

The data in Table 2 will now be used to revise the PLC relation. As in most previous studies we will assume a relation of the form

$$\langle M_v \rangle = A \log P + C(\langle B \rangle_0 - \langle V \rangle_0) + D . \qquad (1)$$

As has been recognized in the past, the sample of galactic Cepheids in clusters is too small and too limited in period range to allow a determination of the slope A and the color term C (Sandage and Tammann 1969; Martin, Warren, and Feast 1979; Cogan 1980, for example). Efforts have been made to improve this situation by searching for more clusters with Cepheids. However, as discussed above in § II, we have selected a limited number of calibrators to avoid the inclusion of spurious information. Thus, we can only determine the zero point and will have to take the other terms from previous studies.

Several previous PLC relations were compared to our data. These are listed in Table 3, where we give the coefficients for each. We also give the quantity $\langle M_v(0.8) \rangle$, which is defined, following de Vaucouleurs (1978*a*), as the absolute magnitude obtained from a relation at $\log P = 0.8$ and $\langle B \rangle_0 - \langle V \rangle_0 = 0.65$. Sandage and Tammann (1969) determined the slope A from a composite period-luminosity relation for external galaxies together with a mean period-color relation. They obtained the color term from theoretical calculations and then found the zero point using open cluster Cepheids. Their zero point is referred to a Hyades modulus of 3.03. Martin, Warren, and Feast (1979) obtained A and C from a study of Cepheids in the Large Magellanic Cloud and set the zero point by reference to open cluster Cepheids and Cepheids with radii measured by the Wesselink method. This study is also consistent with a Hyades distance modulus of 3.03. A problem in comparing their relation with the present calibrators is that the period range of the LMC Cepheids has only a small overlap with that of the open cluster Cepheids. We will, however, assume that the slope and color term remain constant over the entire range. Cogan (1980) also obtained PLC relations from the LMC and

TABLE 3

PLC RELATIONS

	ORIGINAL VALUES				PRESENT VALUES		
SOURCE[a]	A	C	D	$\langle M_v(0.8)\rangle$	D	σ[b]	$\langle M_v(0.8)\rangle$
ST	−3.425	2.52	−2.459	−3.56	−2.37	0.27(0.14)	−3.47
MWF	−3.80	2.70	−2.39	−3.68	−2.21	0.28(0.13)	−3.50
BM	(−5.02)	5	...	...	−2.80	0.20(0.05	−3.57

[a] Sources of PLC relations.—ST, Sandage and Tammann 1969. MWF, Martin, Warren and Feast 1979. BM, Brodie and Madore 1980.

[b] Values in parentheses are with TW Nor omitted.

from the SMC Cepheids. His coefficents for the LMC are similar to those of Martin, Warren, and Feast and are not included in Table 3. Brodie and Madore (1980) carried out numerical simulations of the process of fitting the PLC relation. They found that the color term C was underestimated in the presence of a finite strip width and errors in the intrinsic colors if the fitting was done by least squares. They state that C may be in excess of 5. They do not give an estimate of A, so we determined it from the present data and list it in parentheses in Table 3. It should be noted that this large value of C has been disputed by Feast and Balona (1980), who contend that the value of Martin, Warren, and Feast, derived from the maximum likelihood method rather than least squares, is still valid.

Before deriving the zero point of our PLC relation we should check one final time to see if all the Cepheids in Table 2 are suitable. This was done by plotting $\langle M_v\rangle - C(\langle B\rangle_0 - \langle V\rangle_0)$ against the logarithm of the period. The plot for the Martin, Warren, and Feast relation is shown in Figure 1a. It can be seen that most of the points lie along a line with the appropriate slope. That for CS Vel lies about 3 standard deviations below the mean line, and we conclude it is unlikely to be a member of Ru 79 after all. Fernie and McGonegal (1983) also omitted this star for the same reason. The point representing TW Nor in Ly 6 is about 2 standard deviations below the line. Given the number of clusters involved, it is reasonable to find one that far off, so we will include it. Since this cluster has a relatively large internal error in its distance modulus, it will enter the mean with small weight. Its inclusion changes the mean zero point by only 0.02 mag.

The mean zero points derived by fitting the various relations to the absolute magnitudes of Table 2 are listed in column (6) of Table 3. In forming the means, each cluster was weighted by $1/\sigma^2$, where σ comes from column (7) of Table 2. The three stars in NGC 7790 were each reduced to $\frac{1}{3}$ of this weight since they are not independent. The value in column (7) of Table 3 is the rms scatter of the calibrating stars about the adopted relation. Most of it is contributed by TW Nor; and if that star is omitted, the values are between 0.05 and 0.14 mag and are given in parentheses in the table. Clearly, the scatter is sufficiently small that we can conclude that all three PLC relations are acceptable fits to the data.

The values of $\langle M_v(0.8)\rangle$ implied by our fits to the various PLC relations are given in Table 3. It can be seen that our

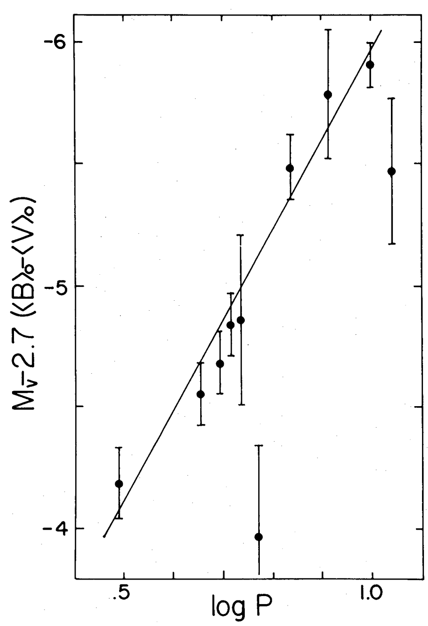

FIG. 1a

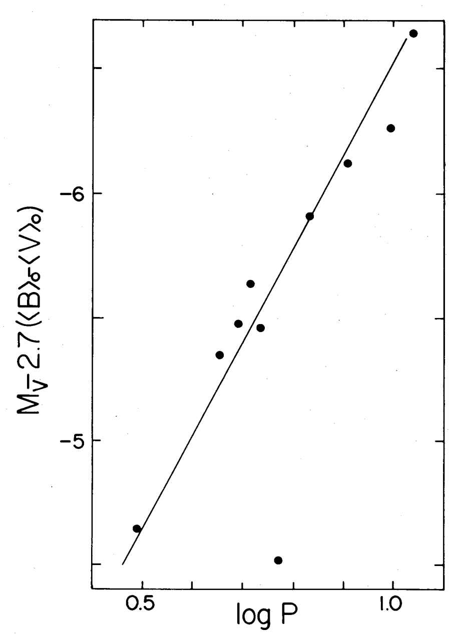

FIG. 1b

FIG. 1.—The absolute magnitude corrected for the color term plotted against the logarithm of the period. The color term of Martin, Warren, and Feast (1979) was used. The solid line has their slope (−3.8) and is fitted to the plotted points. In Fig. 1a the present distance moduli were used while those in Fig. 1b are from the main sequence fitting distance moduli.

calibration implies luminosities lower than that of Sandage and Tammann by 0.09 mag and lower than that of Martin, Warren, and Feast by 0.18 mag. However, if we correct those relations to more recent Hyades distance estimates (about 3.3), the discrepancy with the present determination increases to about 0.4–0.5 mag. These discrepancies will be discussed in the next section in detail.

Since the Martin, Warren, and Feast PLC relation is based on a large body of homogeneous data and individual stars were corrected for reddening, it is likely that it provides the best description of the PLC relation. Thus, we will adopt their values for A and C and our redetermined zero point. Our best estimate of the relation is thus

$$\langle M_v \rangle = -3.80 \log P + 2.70(\langle B \rangle_0 - \langle V \rangle_0) - 2.21 . \quad (2)$$

Strictly speaking, this calibration is only valid in the period range of our calibrating stars, 3–10 days. Any extrapolation to longer periods is only as valid as the slope of the Martin, Warren, and Feast relation and is only as valid as the assumption that the LMC relation can be applied in the Milky Way and elsewhere.

In Figure 1*b* we show the period-luminosity relation for our calibrators using the main-sequence fitting distance moduli. It can be seen that TW Nor in Ly 6 now fits the relation. However, several distances based on main-sequence fitting have been published for this cluster (Madore 1975; Lyngå 1977; van den Bergh and Harris 1976), and they range over an entire magnitude. The good fit in Figure 1*b* depends on choosing the largest of them. Aside from this one star and minor changes in individual points, Figures 1*a* and 1*b* are very similar; the scatter is nearly identical. The only important difference is that the zero points with the present luminosities average 0.55 mag fainter.

McGonegal *et al.* (1983) have derived a period-luminosity law for magnitudes in the J, H, and K bands. Since this near-infrared photometry possesses advantages for distance determination and those same investigators are in the process of obtaining the requisite data for Cepheids in external galaxies, it is worthwhile to tie their photometry to the present calibration. In Figure 2 we have plotted the absolute magnitudes of our calibrators in those three bands against the period. Also plotted are the values used by McGonegal *et al.* as well as fitted relations. It can be seen that the longest-period star in our sample, TW Nor, still falls below the relation defined by the other stars. We have fitted a relation of the form

$$\langle M \rangle = A \log P + D \quad (3)$$

to the data for the three infrared magnitudes. The values of A and D are listed in Table 4. Also listed are the absolute magnitudes at $\log P = 0.8$ both from the present calibration and from that of McGonegal *et al.* It can be seen that the present values are about 0.5–0.6 mag fainter. This is similar to what was found for the M_v calibration and is due to the differences in

TABLE 4

NEAR-INFRARED PERIOD-LUMINOSITY RELATIONS

Band	A	D	$\langle M(0.8) \rangle$	$\langle M(0.8) \rangle$ McGonegal *et al.*
	−2.53	−2.45	−4.47	−5.09
	−3.14	−2.18	−4.69	−5.20
	−3.24	−2.16	−4.75	−5.29

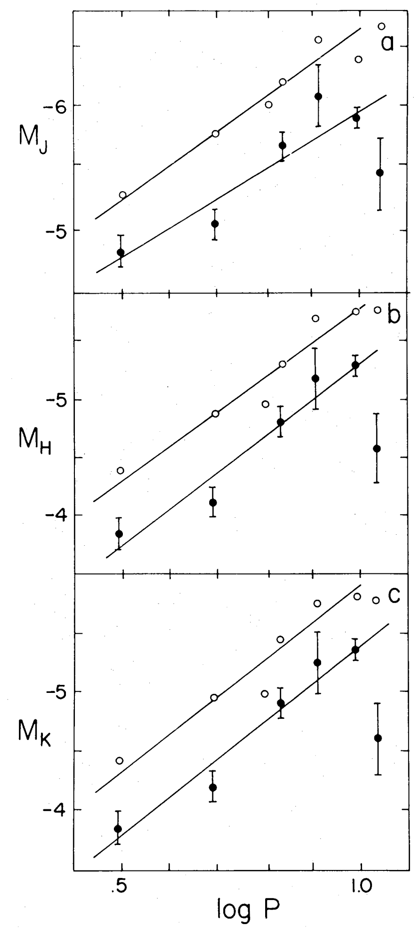

FIG. 2.—The near-infrared period-luminosity relations. Solid circles with error bars represent absolute magnitudes obtained with the present distance moduli and color excesses while the open circles are from McGonegal *et al.* (1983).

the cluster moduli. It should be noted that in a recent study of the Local Group galaxy NGC 6822, McAlary *et al.* (1983) revised the calibration for the magnitude in the H band using more recent unpublished data. Their new relation has a slope which is identical to that obtained here.

V. COMPARISON WITH OTHER LUMINOSITY CALIBRATIONS

a) Calibrations Based on Cepheids in Clusters and Associations

Most modern calibrations of the zero point of the luminosity scale have been based on the Cepheids in open clusters and associations. These include the calibrations of Sandage and Tammann (1969), van den Bergh (1976), de Vaucouleurs (1978*a*), Martin, Warren, and Feast (1979), Caldwell (1983), and Fernie and McGonegal (1983). These various authors have used distance moduli for the clusters and associations which incorporated different assumptions regarding such things as the ratio of selective to total absorption, the difference between the reddening of the B stars and the Cepheids, and the details of the main-sequence fitting. However, these differences are for the most part minor and the values given in column (12) of Table 1 typify those used by all the above authors (when all are corrected to a Hyades distance modulus of about 3.3). The comments which follow are equally relevant to any of the above mentioned calibrations.

The distance moduli from the four-color photometry are smaller on the average than those from main-sequence fitting by 0.50 mag if we exclude Ly 6 from the comparison as discussed in the previous section. The rms scatter about this mean difference is only 0.16 mag, and this indicates good internal consistency among the distance determinations from each method. There is no evidence for a dependence of this difference on such properties as the period of the Cepheids, the distance, the reddening, the richness of the cluster, or the amount of field star contamination. We will therefore assume that the discrepancy is solely in the zero point of the distance scale.

In Table 1 we tabulated the distances of some of the clusters based on MK spectral types. An examination of these shows that all are smaller than the main-sequence fitting distances except for one of the values for M25. Additionally, most are also smaller than the distances from the four-color photometry. Classifications by different investigators often exhibit significant differences as shown by the two values for M25. Thus, while the spectroscopic parallaxes tend to support the current values, this is a fairly weak point.

Efforts have been made recently to carry the *UBV* photometry to very faint apparent magnitudes in M25 (van den Bergh 1978) and NGC 7790 (Pedreros and Madore 1984) in order to better define the zero-age main sequence. M25 is also the cluster with the most extensive four-color photometry while NGC 7790 is the cluster which shows the largest difference between the two types of distance moduli (except for Ly 6). Thus, we will examine these two clusters to gain insight into the discrepancies. The color-magnitude diagram of M25 is plotted in Figure 3*a* using the photoelectric data of Wampler *et al.* (1961) and the photoelectric and photographic data of van den Bergh (1978). The color-magnitude diagram of NGC 7790 shown in Figure 3*b* is based on the photoelectric photometry of Sandage (1958) and the photographic photometry of Pedreros and Madore (1984). The extinction in the field of NGC 7790 is nearly constant while in M25 there is a range in $E(B-V)$ of about 0.2 mag. It can be seen that both color-magnitude diagrams show considerable scatter.

In each of the color-magnitude diagrams we have plotted the zero-age main sequence for the distance moduli and color excesses from the intermediate-band photometry and from the main-sequence fitting to broad-band photometry. There are several effects which introduce uncertainty into the fitting procedure. Each of these will be discussed with reference to Figures 3*a* and 3*b*.

1. *Evolutionary effects.*—In main-sequence fitting it is necessary to avoid using the brightest stars due to stellar evolution. In Figure 3 it is obvious that stars brighter than about $M_v = 1$ may be evolved and should be omitted from the fitting. While in these two clusters there are stars well below that point, most of the clusters' color-magnitude diagrams do not go nearly as faint, and the possibility of errors exists. The difference between the four-color and the main-sequence fitting distances is so constant except for Ly 6, however, that it appears likely that evolution has been accounted for in a generally consistent way and is unlikely to be a serious source of error.

2. *Contamination by field stars.*—As can be seen in Table 1, some stars were rejected as likely nonmembers in every cluster. At fainter magnitudes the number of field stars increases as a result of both the luminosity function and the larger volume of space sampled. On the other hand, the population of cluster stars will rise only because of the luminosity function. Hence, the degree of contamination will be greater further down the main sequence where the fitting was done. An inspection of Figure 3 confirms this expectation and suggests the presence of serious contamination.

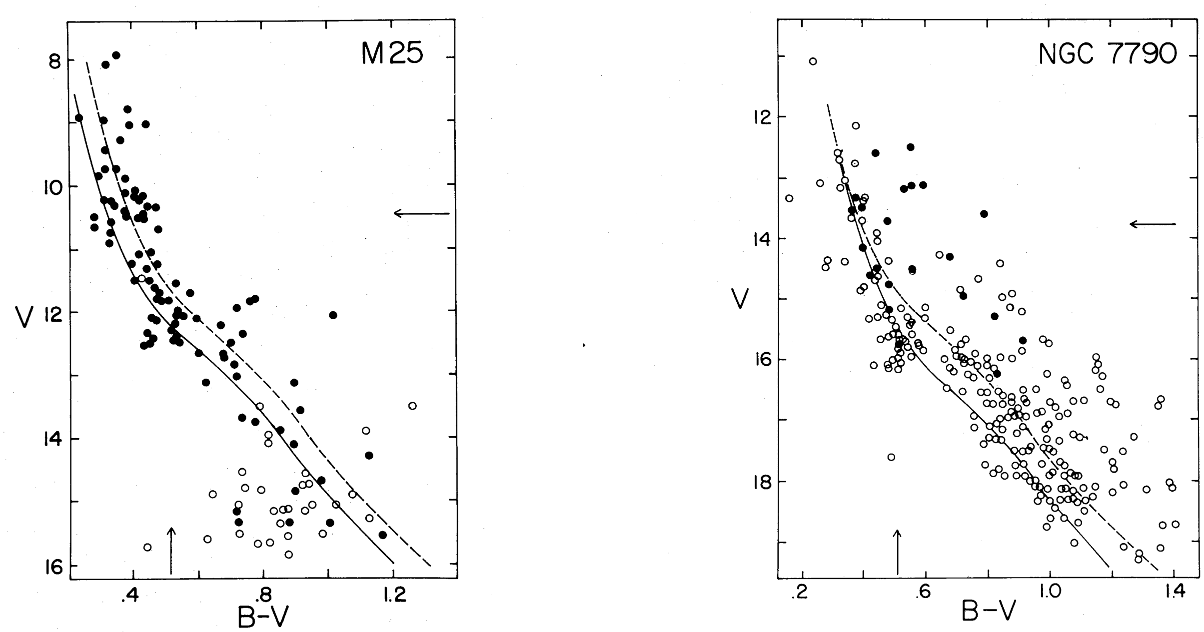

FIG. 3*a* FIG. 3*b*

FIG. 3.—Color-magnitude diagrams for M25 and NGC 7790. Closed circles are photoelectric measurements while open circles are photographic. The solid curve is the Johnson (1963) zero-age main sequence for the distance modulus and color excess obtained from *UBV* photometry (cols. [12] and [13] of Table 1) while the dashed curve is the same curve shifted to the color excess and distance modulus indicated by the four-color and Hβ photometry (cols. [2] and [4] of Table 1). The arrows show the location of $M_v = 0.0$ and $(B-V)_0 = 0.0$ using the present distance moduli and color excesses.

To gain some insight into the problem of field star contamination, a series of artificial color-magnitude diagrams was constructed. The details of this study and a discussion of the general problem of contamination under various circumstances will be given elsewhere. However, it is appropriate to briefly describe the effects found for clusters similar to those studied here.

Seventy-four proper-motion members of the Pleiades were selected to form our artificial cluster. To generate an artificial color-magnitude diagram, the data for a subset of these stars selected at random were corrected to the desired distance and reddening and plotted on a color-magnitude diagram. Field stars were generated randomly in space with color and absolute magnitude distributions corresponding to the known distributions taken from Allen (1973) and added to the diagrams. By altering the number of field stars, we have approximated the situation for the two clusters shown in Figure 3. Some of the artificial color-magnitude diagrams are shown in Figure 4. It can be seen that they are a reasonable approximation to the actual diagrams, and by comparing the uncontaminated diagram (Fig. 4*a*) with those containing field stars it can be seen that they are significantly affected and the amount of contamination increases at fainter magnitudes. The degree of contamination in M25 and NGC 7790 is somewhere between that in Figure 4*b* and Figure 4*c* based on the number of stars rejected in the intermediate band studies.

In Figure 4 we have plotted a fitted main sequence. In each case the fitting was done without knowledge of the properties of the synthetic color-magnitude diagram. The color excesses were, however, known. Thus, this fitting procedure should be a reasonable approximation to the way it is done for real clusters except for uncertainties associated with the reddening. It was found that the clusters in Figures 4*b* and 4*c* yielded smaller distance moduli by 0.15 and 0.34 mag, respectively, than the cluster with no contamination. This is in the opposite sense to that required to explain our discrepancy, but this may have to do with details such as the distance, the color excess, or the statistical treatment of the field stars. In any event, this suggests that errors of 0.1 to 0.3 magnitudes might be introduced by contamination.

3. *Uncertainties in color excesses.*—Since the main sequence is quite steep, errors in the intrinsic colors cause relatively large errors in the fitting of the observed main sequence to the zero-age main sequence (ZAMS). An error of 0.01 mag in $(B-V)_0$ gives rise to a change in the inferred distance modulus of 0.05 mag. Uncertainties in the color excesses are introduced by photometric errors, differing color excesses for early type stars (used in the determination of the color excess) and later type stars (used in the main-sequence fitting), and variations of extinction across the cluster. Color excesses from various sets of photometric data can differ by as much as 0.1 mag (see, for example, Schmidt 1980*c*, 1982*b*), but a few hundredths is more typical. The correction for spectral type between the B stars and the F stars is of the order of 0.01–0.02 mag for $E(B-V) =$ 0.5 mag. If this effect is ignored, main-sequence fitting will give distances which are too large. Thus, it appears that errors of several tenths of a magnitude may be introduced by uncertainties in the color excesses.

4. *Rotation and the presence of binaries.*—It should be noted that the solid curves in Figure 3 are both fitted toward the lower edge of the distribution of apparent main-sequence stars. This is justified on the grounds that binaries will appear above the ZAMS and broaden it. An additional effect which will broaden the main sequence is the effect of rotation. Among B stars, rapid rotators will shift to the right of the main sequence for no rotation by an amount dependent on the rotational velocity (Hardorp and Strittmatter 1968). Unfortunately, the situation is less well known for later spectral types, and we can only assume that it will be similar.

While it is in principle correct to fit to the lower envelope of the main sequence to account for binaries, in practice the validity of this procedure is dependent on how the main sequence was built up. If the clusters which were fitted to the Hyades to form the ZAMS contained binaries or rapid rotators and if the fitting was done to the middle of the distribution, then the present fitting to the lower envelope is incorrect. Additionally, the number of binaries and rapid rotators varies from cluster to cluster. If those used to construct the ZAMS differ from the present clusters, this introduces further uncertainty to the process. It seems reasonable that errors of tenths of a magnitude are possible from this source. Since the tendency is to fit along the lower envelope of the main sequence, this error is likely to be systematic and result in overestimates of distance moduli.

5. *The Hyades distance.*—Most investigators now use a Hyades distance modulus near 3.3. However, there are some who still prefer smaller values (Turner 1979), and it is possible that this is still a source of error at the level of perhaps 0.1 mag.

6. *Metal abundance.*—Van den Bergh (1977) has discussed the effect of metallicity on main-sequence fitting. It is likely

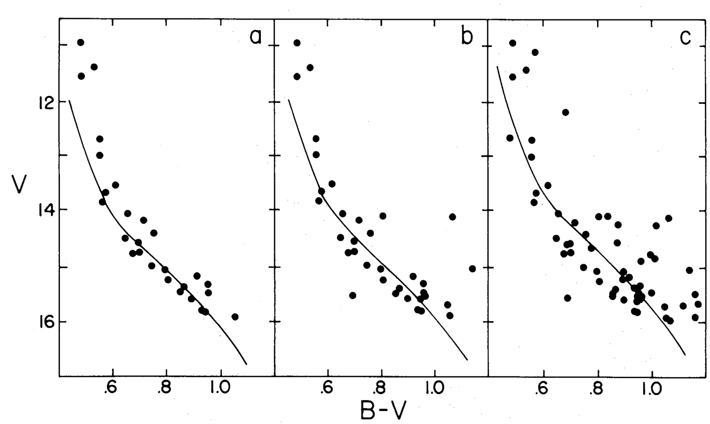

FIG. 4.—Artificial color-magnitude diagrams for a cluster with varying degrees of field star contamination. Fig. 4*a* is the uncontaminated diagram while in Figs. 4*b* and 4*c* 20% and 55% of the stars are nonmembers.

that the Hyades is more metal-rich than the clusters in the present program. The resulting differences in line blanketing will cause the cluster distance modulus to be overestimated by about 0.15 mag according to van den Bergh. If we assume the Hyades is more metal-rich than the program clusters by perhaps 0.2 dex, the models of Mengel *et al.* (1979) can be examined to determine the effect on the luminosity of the ZAMS. It appears that the metallicity of the Hyades will make the ZAMS about 0.1 to 0.2 magnitudes brighter at a given temperature. This will also increase the distances of the program clusters for a total effect of 0.25–0.35 mag.

7. *Helium abundance.*—Strömgren, Olsen, and Gustufsson (1982) discussed the discrepancy in the (c_1, $b - y$) diagram between the sequence of unevolved Hyades stars and the corresponding sequence for unevolved field stars with the same metal abundance. They concluded that the best explanation was that the Hyades has a lower helium abundance by about 0.06 in n_{He}/n_H than field dwarfs. An examination of the zero-age models of Mengel *et al.* (1979) shows that such a decrease in helium will increase the luminosity of the main sequence by about 0.25 mag at a given temperature. If the program clusters have a helium abundance similar to most field dwarfs, the use of the Hyades for the zero point will cause the distances to be overestimated by about that amount.

The various sources of uncertainty are summarized in Table 5. A positive entry indicates that the distance modulus will be overestimated from main-sequence fitting. None of these effects appears likely to explain the entire discrepancy between the intermediate-band and the broad-band distance moduli. However, it is likely that several of them operate, and the combined uncertainty they introduce is certainly large enough to account for it. This is particularly true since several are in the right sense. Further work is needed to clarify the situation, but there is no reason to prefer the distance obtained from main-sequence fitting over those obtained from the intermediate-band photometry; rather, it appears that the intermediate-band photometry is likely to be more reliable since it avoids some of the problems with main-sequence fitting.

b) *Distances Based on the Barnes-Evans-Parsons Method*

Barnes, Evans, and Parsons (1976) developed a method of determining simultaneously the distance and radius of a Cepheid using the surface brightness inferred from the color and the surface displacement calculated from the radial velocity curve. This method is independent of other distance determinations and in principle is a powerful way to establish the distances and luminosities of these stars. Barnes *et al.* (1977) applied the method to nine Cepheids for which the requisite data existed and found distances which were consistent with those predicted by the Sandage and Tammann (1969) PLC relation (based on the old Hyades distance). The surface brightness relation was subsequently refined by Barnes, Evans, and Moffett (1978). However, in the most recent discussion of this work, Barnes (1979) did not use the refined relation. Rather, he determined the slope of the relation using the Cepheids themselves. The resulting distances are larger than the previous values and now match the Sandage and Tammann scale when it is corrected to the new Hyades distance. Since one of the strengths of this method is the fact that the same surface brightness relation applies to stars of all gravities (Barnes, Evans, and Moffett 1978), it is disturbing that the Cepheids do not seem to conform to that relation.

Clearly, in assessing the distances determined by this method an important ingredient is the surface brightness relation. The difference between the distances of Barnes *et al.* (1977) and those of Barnes (1979) (0.3 mag) were produced by altering the surface brightness by between 0.001 at $(V-R)_0 = 0.4$ and 0.012 at $(V-R)_0 = 0.8$. The surface brightness for non-Cepheids of Barnes, Evans, and Moffett (1978) differs from both these relations by much more. It matches them near $(V-R)_0 = 0.5$ but is less by 0.07 at $(V-R)_0 = 0.8$. Additionally, an inspection of the scatter in the relation for Cepheids (Barnes 1979) shows that it is uncertain at a level at least as large as the difference between the two relations used in Barnes *et al.* (1977) and Barnes (1979). Thus, it appears that the results of this work are not yet capable of accuracies better than several tenths of a magnitude in the distance modulus. We conclude that the difference between the present scale and that of Barnes (1979), about 0.4 mag, is within the current uncertainty of the Barnes-Evans-Parsons method. Fortunately, this situation will be improved in the near future. Barnes and Moffett (private communication) have obtained accurate light and color curves simultaneously with velocity curves for a large sample of Cepheids. This will allow them to better define the surface brightness relation for the Cepheids and thus eliminate the present criticism of this method. It will also provide many more stars to calibrate the PLC relation.

c) *Statistical Parallaxes*

Clube and Dawe (1980) carried out a statistical analysis of the space motions of Cepheids. This study gave absolute magnitudes which were fainter than those of the Sandage and Tammann (1969) scale by 0.15 ± 0.3 mag. This is very good agreement with the present scale, but unfortunately the errors of the statistical analysis do not place strong constraints on the scale. Thus, we regard this as very weak support for the present PLC relation.

VI. CONSEQUENCES OF THE REVISION

a) *Pulsation Theory*

The luminosities of the Cepheids were obtained from the absolute magnitudes of Table 2 and the bolometric corrections of Sandage and Gratton (1963). The scale of Pel (1978) was used for the effective temperatures. The radii of the stars were next determined from these values. Cox (1979) gave an expression for the pulsation constant, Q_0, in terms of the mass, radius, luminosity, and effective temperature. The luminosity, temperature, and radius were used in this expression to calculate Q_0 and thence the period for various trial masses. The mass which reproduced the actual period of the star was taken to be the pulsational mass. Evolutionary masses were calculated with the mass-luminosity relation of Becker, Iben, and Tuggle (1977) for $Y = 0.28$ and $Z = 0.02$.

TABLE 5

SOURCES OF ERROR IN MAIN-SEQUENCE FITTING

Source	Estimated Maximum Effect on Distance Modulus (mag)
Evolution	0.0
Contamination	−0.3 to −0.1
Uncertainties in color excesses	−0.3 to +0.4
Rotation and binaries	+0.3
Hyades distance	+0.1
Metallicity	+0.25 to +0.35
Helium abundance	+0.25

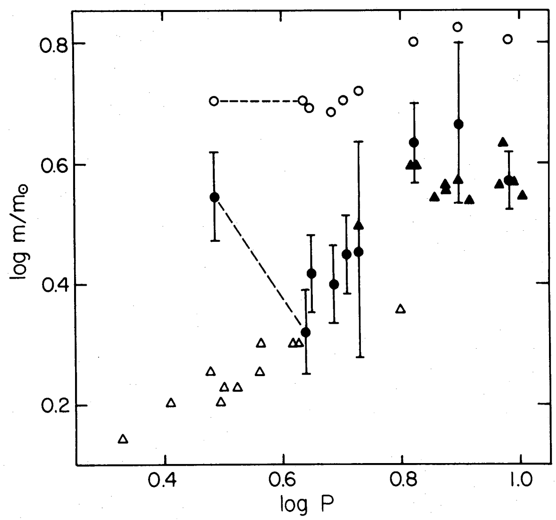

FIG. 5.—Cepheid masses plotted against period. Filled circles represent pulsational masses of the cluster Cepheids while the open circles are their evolutionary masses. The pairs of points connected by dashed lines represent EV Sct under alternate assumptions regarding its pulsation mode. Open triangles indicate the masses of beat Cepheids; filled triangles represent bump masses.

The mass estimates for the cluster Cepheids are plotted against period in Figure 5. Filled circles represent the pulsational masses. The error bars were calculated from the uncertainties in the absolute magnitudes given in Table 2. Open circles denote the evolutionary masses, and the errors for them are approximately one-fifth as large as for the pulsational masses. It can be seen that the pulsational masses follow a well-defined sequence which is between 40% and 70% of the evolutionary masses; the mass discrepancy has been reinstated. For comparison we also show the masses of the bump and beat Cepheids (from Fricke, Stobie, and Strittmatter 1972 and Cox 1982, respectively) as filled and open triangles, respectively. It can be seen that these masses fall along a sequence which is nearly identical to that of the pulsational masses.

There is one star which departs from the trend. The pulsational and evolutionary masses of EV Sct (at $\log P = 0.49$) both lie above the sequences of the other stars. A possible explanation is that it is an overtone pulsator. If this is the case, its fundamental period would be 4.37 days ($\log P = 0.64$) rather than 3.09 days. The pulsational mass for that period is less and fits in with the other pulsational masses. Moving the evolutionary mass to the longer period also improves its agreement with the others. The points for this star under the alternate assumptions are connected by dotted lines in Figure 5.

In the derivation of the P-L relation we assumed that EV Sct was a fundamental pulsator. However, an examination of Figure 1*a* indicates that if we shift the point representing EV Sct from $\log P = 0.49$ to $\log P = 0.64$, a slightly steeper line will fit the points as well as that shown. This is not true when the main-sequence-fitting moduli are used (Fig. 1*b*). However, the problem there rests with the three stars from NGC 7790. In Figure 1*b* these three stars are all too bright compared with CV Mon and U Sgr which have only slightly longer periods. If we trust the distance for M25 over that for NGC 7790 and ignore those three points in Figure 1*b*, we can again get a good fit with EV Sct moved to the longer period. An examination of the infrared P-L relations (Fig. 2) shows that if EV Sct is moved to $\log P = 0.65$, fits can be made which are better than those shown. Thus, there is no evidence to contradict the conclusion the EV Sct is an overtone pulsator. It should be studied further in that light.

Most attempts to rectify the discrepancy between pulsational and evolutionary masses have centered on finding ways to increase the pulsational masses. This was in part motivated by the recognition that alterations in the luminosities and temperatures which were within the uncertainties could force agreement between M_Q and M_{ev}. However, as described in the Introduction, a continuing problem has been the lack of agreement between M_Q and M_{ev} on one hand and the bump and beat masses on the other. With the present luminosity scale the pulsational masses are in reasonable agreement with the bump and beat masses. The various results of pulsation theory are thus consistent, and the discrepancy is entirely between evolution theory and pulsation theory. Previous attempts to bring the various masses calculated from pulsation theory individually into agreement with the evolutionary masses may have been misdirected. It appears that the disagreement is more fundamental than this and must be rectified by improvements in one or the other of the two theories. Given that the pulsation theory relies on less physics and is less sensitive to computational details than evolution theory, it seems possible that the stellar evolution models may require reevaluation.

b) The Cosmic Distance Scale

The Cepheids are a basic distance indicator in the Local Group, and obviously altering the calibration will have an important effect on the cosmic distance scale. However, different investigators have used a variety of indicators, and the weight of the Cepheids in the result varies (see van den Bergh 1977 for a discussion of this). A full discussion of the effect of the present revision on this field is beyond the scope of this paper. Nonetheless, it is in order to show that the result is not inconsistent with other primary distance indicators.

In Table 6 we list distance estimates for the Large Magellanic Cloud based on a variety of indicators from two papers (van den Bergh 1976; de Vaucouleurs 1978*b*). Also given are the Cepheid moduli from the present calibration. The distance modulus of Martin, Warren, and Feast (1979) was adjusted to the present zero point to obtain the *UBV* distance modulus, while the *H*-band photometry of McGonegal *et al.* (1982) was used together with equation (3) to derive the near-infrared dis-

TABLE 6

DISTANCE ESTIMATES FOR THE LARGE MAGELLANIC CLOUD

METHOD	TRUE DISTANCE MODULUS		
	vdB	dV	Present
Cepheids *UBV* photometry	18.70	18.27	18.51
H band photometry (MMMM Data)	...	...	18.17
H band photometry (LS data)	...	...	18.07
Novae	...	18.46	...
RR Lyrae	18.43	18.17	...
A, B supergiants	...	18.32	...
Eclipsing binaries	...	18.36	...
Supernova remnants	18.36	...	...
Globular cluster giants	18.50	...	...
Hβ, Hγ	18.20	...	...
Spectral gradients	18.58	...	...
MK classifications	18.70	...	...
Mean, omitting Cepheids	18.46	18.33	...

tance modulus. Laney and Stobie (unpublished work cited by Feast 1983) have also carried out H band photometry of LMC Cepheids. The individual observations are not available, but according to Feast they fit a relation of the form

$$\langle H_0 \rangle = -3.34 \log P + 16.19 . \quad (4)$$

Since the slope of this relation is different from the slope we have derived, the period at which we make the comparison is crucial. In the absence of any information on the periods of the stars observed by Laney and Stobie, we will assume a mean period of about 31 days ($\log P = 1.5$), and the resulting distance modulus, 18.07, is in agreement with that obtained from the McGonegal *et al.* photometry. However, both are surprisingly small compared with that obtained from the *UBV* data (18.51).

A further check on the consistency of distances from *UBV* photometry and infrared photometry can be made by considering data for NGC 6822. Kayser (1967) obtained B and V light curves for Cepheids in that galaxy. Using the period-color relation of van den Bergh (1976), we can convert our adopted PLC relation to the following P-L relation:

$$\langle M_v \rangle = -2.5 \log P - 1.5 . \quad (5)$$

When this is applied to all the Cepheids in NGC 6822 with periods less than 50 days, we obtain a mean distance modulus of $\langle V - M_v \rangle = 24.19 \pm 0.08$ (standard error of the mean). Van den Bergh and Humphreys (1979) estimated that the reddening of these Cepheids was likely to be about $E(B-V) = 0.42$, so the true distance modulus becomes 22.8. On the other hand, an application of the present H-band calibration to the data of McAlary *et al.* (1983) yields a mean true distance modulus of 22.86 ± 0.08, which is in good agreement.

We conclude that the present calibration of the Cepheid luminosity scale does not introduce serious discrepancies when the Cepheids are compared with other distance indicators.

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